Exam program (Version 1)

1. Proofs of the formulas for numbers of permutations and combinations with and without repetitions.

2. Newton's binomial theorem. Multinomial coefficient and the multinomial formula.

3. Combinatorial identities (6 identities).

4. Principle of inclusion–exclusion. Formula for the number of derangements of n elements.

5. Möbius function. Sum of values of the Möbius function over the divisors of a number, and over the divisors whose prime factorizations contain an even number of prime factors.

6. Theorem on the Möbius inversion formula. Application of the Möbius inversion formula to counting the number of cyclic sequences. Cyclic shift operation on linear sequences. Period of a linear sequence. Properties of the period. General form of a sequence with period d and length n.

7. Möbius function on partially ordered sets. Coincidence of the Möbius function $\mu(1, n)$ on the poset $(\mathbb{N}, |)$ with the classical Möbius function $\mu(n)$ on $\mathbb{N}$ (for all n). General Möbius inversion formula for partially ordered sets.

8. Reproof of the inclusion–exclusion formula using the Möbius inversion formula on a partially ordered set.

9. Asymptotic value of the binomial coefficient for $k = o(\sqrt{n})$.

10. Compositions of an integer n. Recurrence relations.

11. Partitions of an integer n. Recurrence relations.

12. Young diagrams. Euler's theorems on the equality of the numbers of unordered partitions of an integer n into k or fewer parts and into parts not exceeding k (three theorems).

13. Formal power series $(1-x)(1-x^2)(1-x^3)\cdots$. Euler's theorem on the coefficients of this series. Verification of Euler's theorem for $k = 1, 2$. Theorem on the number of partitions into an even and an odd number of distinct summands (statement).

14. Linear recurrence relations with constant coefficients. Characteristic equation. General formula for a recurrence of order n (statement; proof for special case $n = 2$). Formula for Fibonacci numbers. Derivation of special case $n = 2$ from the general statement.

15. Formal power series and operations on them. Definition of the inverse series. Example of division by columns. A combinatorial identity obtained using the formal power series $\dfrac{1}{(1-x^2)^2}$.

16. Computation of the sum $\displaystyle\sum_{k=0}^{n} k^2 \binom{n}{k} \left(\frac{1}{17}\right)^k$.

17. Definition of the radius of convergence of a power series. Cauchy–Hadamard theorem on the convergence of power series (statement). Examples illustrating the theorem (three examples showing that convergence or divergence may occur on the boundary).

18. Fibonacci numbers and their generating function. Radius of convergence. Value of $\displaystyle\sum_{n=0}^{\infty} \frac{F_n}{2^n}$.