

Probability. Combinatorics. Geometric probability. Bernoulli scheme. Conditional probability. Bayes' Theorem.

Dateline: 23:59, Sunday, September 27, 2026

1. **(2 pts)** Harry, Hermione and Ron and other 13 students are studying together. All the students were randomly divided into 4 teams of 4 people each. What is the probability of Harry, Hermione and Ron being in the same team?
2. **(3 pts)** A box contains 100 balls, of which m are red. Suppose that the balls are drawn from the box one at a time, at random, without replacement. Determine
 - (a) the probability that the first ball drawn will be red;
 - (b) the probability that the 50th ball drawn will be red;
 - (c) the probability that the last ball drawn will be red.
3. **(6 pts)** Suppose there are n theatre seats in a row. People are seated in a random manner.
 - (a) If n people occupy n seats in a row what is the probability that two particular people A and B will be seated next to each other?
 - (b) If k people are seated in a row ($k < n$) what is the probability that the people will occupy k adjacent seats in the row?
 - (c) k colleagues came to the theatre but they don't like each other are seated in a row. If each of them sits at least 2 seats apart from any other colleague, then they all will enjoy the performance. What is the probability that all these k colleagues enjoyed the performance?
4. **(5 pts)** A closet contains n pairs of boots. $2r$ boots are randomly selected from the closet. What is the probability that among the selected boots
 - (a) There are not complete pairs.
 - (b) There is exactly one complete pair.
5. **(5 pts)** Two M -sided dice are thrown. Find the probability that the sum of the two numbers obtained is equal to i .
6. **(4 pts)** A group of n friends is hanging out on a street. There are m bars on the street. Each friend randomly walks into some bar. What is the probability that there is at least one of the friends in every bar (no bar is left empty)?
7. **(6 pts)** An exam consists of five problems, each worth between 0 and 10 points, with only integer scores allowed. Two students take the exam and, surprisingly, both receive exactly 33 points in total. Assume that, among all possible ways of distributing 33 points across the five problems subject to the 10-point maximum per problem, every score distribution is equally likely for each student. What is the probability that the two students received exactly the same score on every single problem?
8. **(6 pts)** The black and white kings are on the first and third rows, respectively, of a chess board. The queen is placed at random in one of the free squares of the first or second row. Find the probability that the position for the black king becomes checkmate if the positions of the kings are equally probable in any squares of the indicated rows.

9. **(3 pts)** Two positive numbers X and Y are taken at random. Both of them do not exceed 2. Find the probability that the product XY is at most 1, and the quotient Y/X is at most 2.
10. **(6 pts)** Pope Julius II hires Michelangelo and his several assistants to paint a Sistine Chapel ceiling. If an artist works for x weeks a month (x is a real value), he paints x^2 square meters of the ceiling. Each artist chooses his workload for the next month at random and it can take values from 0 to 4 weeks per month.
- What's the probability that two artists will paint less than 4 m^2 of the ceiling next month?
 - What's the probability that two artists will paint more than 9 m^2 of the ceiling next month?
 - What's the probability that three artists will paint more than 16 m^2 of the ceiling next month?

After a few months have passed the Pope was dissatisfied by the pace of work and obliged each artist to work at least 2 weeks per month. Under new circumstances what will be the probabilities in 10a, 10b, 10c?

11. **(4 pts)** Nickolay lives in Saint-Petersburg where it's either raining or snowing or windy (so the weather is just bad) 219 days per year. He always takes a taxi to get to work. If the weather is bad there's 60% chance there will be high demand for taxi. If the weather is fine there's 20% chance the demand will be high. If the weather is bad and the demand for taxi is high there's 70% chance Nickolay is late for work. On the other hand, there's 10% chance Nickolay is late (if the weather is fine and the demand is moderate or low). In all other situations there's 25% chance Nickolay is late.
- What's the probability that the weather is fine and there's high demand for taxi and Nickolay is on time for work?
 - What's the probability that Nickolay is late?
 - If Nickolay was late someday, what's the probability the weather was bad that day?
12. **(5 pts)** A box contains r red balls and b blue balls. One ball is selected at random and its color is observed. The ball is then returned to the box and k additional balls of the same color are also put into the box. A second ball is then selected at random, its color is observed, and it is returned to the box together with k additional balls of the same color. Each time another ball is selected, the process is repeated. If four balls are selected, what is the probability that the first two balls will be red and the third and forth ball will be blue?
13. You meet a man with a coin on the street and he suggests playing such a game: a coin is flipped again and again until there is Heads-Heads (two Heads in a row) or Heads-Tails (Tails after Heads). In case of Heads-Heads the man gives you ₱1000, otherwise you give ₱1000 to the man.
- (2 pts)** Is the game fair assuming the coin is fair? Justify your answer by calculating the probability of you winning the game.
 - (4 pts)** Will the answer change if the rules will change: you win if a combinations Heads-Heads appears first, but the man wins if Tails-Heads appears first (even if the coin is assumed fair)? Justify your answer by calculating the probability of you winning the game.

14. **(6 pts)** From an urn containing a white and b black balls, two players take turns drawing balls. The winner is whoever chooses the white ball first. Find the probability that the first player wins in cases where the balls are drawn
- (a) under the scheme of equiprobable choice with return;
 - (b) under the scheme of equiprobable choice without replacement.