Problem set 1 - Probability distribution. CDF. PDF

1. Let ξ be a random variable with the following PDF

$$p_\xi(x) = \begin{cases} 0 & , x < 1; \\ \frac{c}{x^4} & , x \geq 1. \end{cases}$$

   Find **a)** Value of c. **b)** CDF of ξ. **c)** $\mathsf{P}(\xi = 2)$. **d)** $\mathsf{P}(\frac{1}{2} < \xi < 3)$.

2. Let $\xi \sim \mathcal{N}(2, 4)$ and $\eta = 3 - 2\xi$.

   a) Find $\mathsf{P}(\xi > 1)$

   b) Find $\mathsf{P}(-2 < \eta < 1)$.

3. The Probability density function (PDF) of a random variables ξ is given by $p(x)$. Find the corresponding CDF for

   (a) $p(x) = \frac{\theta}{\pi(\theta^2+(x-x_0)^2)}$ (Cauchy distribution with scale parameter θ and location parameter x_0);

   (b) $p(x) = xe^{-x}I(x > 0)$ (Gamma distribution with parameters 2,1);

   (c) $p(x) = \frac{1}{b-a}I(a \leq x \leq b)$ (uniform distribution in $[a, b]$);

   (d) $p(x) = k(x-1)^{k-1}I(1 \leq x \leq 2)$, $k \in \mathrm{N}$.

4. Let ξ_1 and ξ_2 be random variables with uniform distribution in $[0, 1]$. Let ξ_3 have exponential distribution with parameter $\lambda > 0$. ξ_1, ξ_2 and ξ_3 are mutually independent. Find

   (a) $\mathbb{P}\{\xi_1 + \xi_3 \leq 3\}$;