

Problem set 3- Independence. Convolution formula

1. Let ξ, η be independent random variables with uniform distribution in $[0, a]$. Find the density of the random variables $\xi + \eta, \xi - \eta, \xi\eta, \xi/\eta$.
2. Let ξ and η be independent random variables, such that $p_\xi(x) = e^{-x}I(x \geq 0)$ и $p_\eta(y) = e^{-y}I(y \geq 0)$. Find $p_{\xi+\eta}(z)$.
3. Let X and Y be independent random variables. Find the probability that segments with lengths X, Y and 1 can form a triangle if X has uniform distribution on $[0, 1]$ and Y has exponential distribution with parameter 1.
4. Let ξ_1 and ξ_2 be independent random variables. Find $p_{\xi_1+\xi_2}$ if
 - (a) $\xi_i \sim \mathcal{N}(a_i, \sigma_i^2), i = 1, 2;$
 - (b) $\xi_i \sim \Gamma(\alpha_i, \lambda), i = 1, 2.$
5. Let ξ_1, \dots, ξ_n be independent random variables with standard normal distribution. Find that distribution of $\xi_1^2 + \dots + \xi_n^2$.