Problem set 4 - Expectation of Continuous distributions

1. Find the Expected value and Variance of a random variable with the following distributions:

   (a) $\xi \sim U[a, b]$.

   (b) $\xi \sim \mathcal{N}(a, \sigma)$

   (c) $\xi \sim \Gamma(\alpha, \beta)$

   (d) ξ has Beta distribution with parameters (α, β).

   (e) $\xi \sim \text{Exp}(\lambda)$.

2. Let $\xi_1, \xi_2, \ldots, \xi_n$ be independent random variables with uniform distribution in $[0, 1]$. Find $\mathsf{E}\xi_{(k)}$ and $\mathsf{Var}\xi_{(k)}$.

3. The PDF of η is given by

$$p_\eta(x) = \begin{cases} A\sin 2x & , x \in [0, \frac{\pi}{2}]; \\ 0 & , \text{otherwise.} \end{cases}$$

   Find (a) Value of A. (b) CDF of η, (c) $\mathsf{E}\eta$, $\mathsf{Var}\eta$. (d) $\mathsf{P}(-\frac{\pi}{6} \leq \eta \leq \frac{\pi}{6})$.

4. Let ξ have the following CDF:

$$F_\xi(x) = \begin{cases} 0 & , x < -2; \\ \frac{1}{5} & , -2 \leq x < 1; \\ \frac{x^2}{4} & , 1 \leq x < 2; \\ 1 & , x \geq 2. \end{cases}$$

   Find $\mathsf{E}\xi$ and $\mathsf{Var}\xi$.

5. The coordinates of two points on the line are independent and have an uniform distribution on $[0, 1]$. Find the expected value and the variance of the distance between these two points.

6. The joint PDF of a random vector (ξ, η) is given by

$$p_{\xi,\eta}(x, y) = \begin{cases} Axy & , (x, y) \in D; \\ 0 & , (x, y) \notin D. \end{cases}$$

   where $D : \{x \geq 0, y \geq 0, x + y \leq 2\}$. Find

   (a) The value of A.

   (b) $\mathsf{P}((\xi, \eta) \in G)$, where $G : \{0 \leq x \leq 1, 0 \leq y \leq 1\}$.

   (c) $p_\xi(x)$, $p_\eta(y)$. Are ξ and η independent random variables?

   (d) $\mathsf{E}\xi$, $\mathsf{E}\eta$.

   (e) $\mathsf{Var}\xi$, $\mathsf{Var}\eta$.

   (f) $\mathsf{Cov}(\xi, \eta)$ and $\mathsf{Corr}(\xi, \eta)$