Syllabus - Probability Theory by *Carlos Buitrago*

1. Basics of combinatorics. Rule of sum and rule of product. Pigeon-hole principle. Permutations and combinations. Binomial and Multinomial theorems. Combinatorial identities (7 formulas).

2. Inclusion-exclusion principle. Random experiments and notion of probability. Probability in nature. Probability triple $(\Omega, \mathcal{F}, \mathrm{P})$. Discrete sample space. Classical probability model. Problem on getting a lucky ticket in a tram in Dolgoprudny.

3. Bernoulli scheme. k successes in n trials. Sum of all outcomes gives 1. Geometric probability. Probability of two people meeting. Bertrand's Paradox.

4. Definition and examples of σ-algebras. Algebra which is not a sigma-algebra. Measurable space. Definition and properties of the Probability measure with proofs.

5. Continuity of the probability measure. Probability of union of intersecting events and its upper bound. Conditional probability. Theorem of multiplication. Independence of events (pairwise and mutually). Example of pairwise but not necessarily mutually independent events. Example on the necessary conditions for mutual independence.

6. Partition of Ω. Total Probability Formula. Bayes Theorem. Monty-Hall problem. Example with blood test. Problem on choosing an easy question at the exam. Problem of an old lady in a plane. Notion of random variables. Number of successes in Bernoulli scheme as a random variable.

7. Probability distributions. Cumulative probability distribution. Uniqueness of CDF. Properties of CDF. Types of probability distributions. Discrete probability distributions. Examples: Bernoulli, Binomial, Uniform, Geometric, Poisson. Poisson limit theorem. Absolutely continuous distributions. Properties of PDF. Examples: Uniform, Exponential, Normal, Gamma, Beta.

8. Probability distributions in $\mathbb{R}^n$. CDF in $\mathbb{R}^n$. Properties of CDF in $\mathbb{R}^n$. Marginal distributions. Independence of distributions. Independence of CDFs. Absolutely continuous distributions in $\mathbb{R}^n$. PDF of cartesian product of distributions. Properties of PDF in $\mathbb{R}^n$.

9. Random element (variable and vector). $\mathcal{F}_\xi$ is a σ-algebra. Borel functions. Borel functions of random vectors. Measurability criterion. Theorem on the components of a random vector. $\mathbb{P}_\xi$ is a probability distribution.

10. Independence of random vectors. Independence random variables and Borel functions. PDF of a random vector with independent components. Criterion for independence of random variables using CDF.

11. Convolution formula for the sum, difference, multiplication and division of random variables. Chi-squared distribution. Sum of independent random variables with Binomial distribution. Sum of independent random variables with Poisson distribution .

12. Definition of $\mathsf{E}\xi$ as the Lebesgue integral of ξ. Properties of the Expected value with proofs(5). Theorem on the change of variable. Markov's, Chebyshev's and Jesen's inequalities.

13. Expected value of variance of some distributions: Bernoulli, Binomial, Poisson, Geometric, Uniform continuous, Exponential, Normal, Gamma, Beta, Couchy. k-th moment of normal random variable.

14. Variance. Properties with proofs (3). Covariance . Example of dependent random variables with zero covariance. Properties of covariance (5). Variance of sum of n random variables. Correlation coefficient, definition and meaning. $|\mathsf{corr}(\xi, \eta)| \leq 1$. Proof using variance.

15. Weak Law of large numbers. General form of WLLN. WLLN proves that probability is frequency.

16. De Moivre-Laplace Local limit Theorem. De Moivre-Laplace Integral limit Theorem. Example.

Список литературы

[1] Introduction To Probability - Joseph K. Blitzstein, Jessica Hwang.

[2] Introduction to probability for Data Science - Stanley H. Shan.

[3] Probability (Graduate Texts in Mathematics) 2nd Edition - Albert N. Shiryaev.