

Problem set 11 - Expected value and Variance of Continuous Distributions

- (on *Moire-Laplace*) 1800 dice are thrown. Find an approximate value for the probability that the total number of occurrences of 2 and 6 is not less than 620.
- (on *Poisson limit Th.*) When typing, the stenographer makes a mistake in the character with probability of 0.0005. Find an approximate value for the probability that, when typing 10.000 characters, the stenographer will make a mistake no more than three times.
- Find the Expected value and Variance of a random variable with the following distributions:
 - $\xi \sim \mathcal{N}(a, \sigma)$
 - $\xi \sim \Gamma(\alpha, \beta)$
 - ξ has Beta distribution with parameters (α, β) .
- In a triangle with sides 3, 4 and 5, a random point X is selected. Let ξ be a random variable equal to the sum of the lengths of the distances from X to each of the sides of the triangle. Find $E\xi$.
- Let $\xi_1, \xi_2, \dots, \xi_n$ be independent random variables with uniform distribution in $[0, 1]$. Find $E\xi_{(k)}$ and $\text{Var}\xi_{(k)}$.
- Let $\xi \sim \mathcal{N}(0, \sigma^2)$, $k \in \mathbb{N}$. Find $E\xi^k$ and $E|\xi|^k$.
- Let ξ be a random variable with CDF $F(x) = xI(\frac{1}{4} \leq x \leq \frac{1}{2}) + I(x \geq \frac{1}{2})$. Find $E\xi$ and $\text{Var}\xi$.
- Are there ξ, η such that they have standard normal distribution and $\text{cov}(\xi, \eta) = 0$, but they are not independent?
- The PDF of η is given by

$$p_\eta(x) = \begin{cases} A \sin 2x & , x \in [0, \frac{\pi}{2}]; \\ 0 & , \text{otherwise.} \end{cases}$$

Find (a) Value of A . (b) CDF of η , (c) $E\eta$, $\text{Var}\eta$. (d) $P(-\frac{\pi}{6} \leq \eta \leq \frac{\pi}{6})$.

- The coordinates of two points on the line are independent and have an uniform distribution on $[0, 1]$. Find the expected value and the variance of the distance between these two points.
- A sniper in the shooting range shoots at the “quarter circle”, that is, at the area $D = \{(x, y) : x^2 + y^2 < 1, x > 0, y > 0\}$. The random vector (ξ, η) represents the hit point of the shooter and it has uniform distribution in D . Find $\text{cov}(\xi, \eta)$.
- Let the random vector (X, Y) have the following density

$$p_{X,Y}(x, y) = \frac{1}{2\pi\sqrt{1-\rho^2}} e^{-\frac{1}{2(1-\rho^2)}(x^2 - 2\rho xy + y^2)}$$

Find the covariance matrix of (X, Y) and the distribution of X .

Extra Problems

13. Find the Expected value and Variance of a random variable with the following distributions:

(a) $\xi \sim U[a, b]$.

(b) $\xi \sim \text{Exp}(\lambda)$.

(c) $\xi \sim \text{Laplace}(0, \frac{1}{\lambda})$.

14. Consider the random variables $\xi \sim \mathcal{N}(0, 1)$ and $\eta \sim \text{Ber}(1/2)$, such that $\xi - \eta$ and η are independent. Find $\mathbb{E}e^{\xi + \eta}$.

15. Let ξ have the following CDF:

$$F_{\xi}(x) = \begin{cases} 0 & , x < -2; \\ \frac{1}{5} & , -2 \leq x < 1; \\ \frac{x^2}{4} & , 1 \leq x < 2; \\ 1 & , x \geq 2. \end{cases}$$

Find $\mathbb{E}\xi$ and $\text{Var}\xi$.

16. Let $\xi_1, \xi_2, \dots, \xi_n$ be i.i.d.r.v. Find $\mathbb{E}\frac{\xi_1}{\xi_1 + \dots + \xi_n}$.

17. Let ξ_1 and ξ_2 be independent random variables with distribution $\text{Exp}(\lambda)$. Find $\text{cov}(\frac{\xi_1}{\xi_1 + \xi_2}, \xi_1 + \xi_2)$.

18. The joint PDF of a random vector (ξ, η) is given by

$$p_{\xi, \eta}(x, y) = \begin{cases} Axy & , (x, y) \in D; \\ 0 & , (x, y) \notin D. \end{cases}$$

where $D : \{x \geq 0, y \geq 0, x + y \leq 2\}$. Find

(a) The value of A .

(b) $\mathbb{P}((\xi, \eta) \in G)$, where $G : \{0 \leq x \leq 1, 0 \leq y \leq 1\}$.

(c) $p_{\xi}(x), p_{\eta}(y)$. Are ξ and η independent random variables?

(d) $\mathbb{E}\xi, \mathbb{E}\eta$.

(e) $\text{Var}\xi, \text{Var}\eta$.

(f) $\text{Cov}(\xi, \eta)$ and $\text{Corr}(\xi, \eta)$