Problem set 2 - Classical probability

1. Suppose n friends are sitting around a table in random order. What is the probability that A sits next to B?

2. In a box there are 10 white balls and 6 black. 4 balls are chosen randomly from among them without return. What is the probability that, among the chosen balls

   (a) there is at least one black ball

   (b) there are exactly two black balls

3. Two students alternately toss a coin. The winner is the one in getting tails first. Find the probability of winning for each player. Describe the probabilistic space of this problem.

4. A closet contains n pairs of boots. $2r$ boots are randomly selected from the closet. What is the probability that among the selected boots

   (a) There are not complete pairs.

   (b) There is exactly one complete pair.

5. Some residents of Dolgoprudny consider a tram ticket "special" if the sum of the first three digits, of its six-digit number, is equal to the sum of the last three digits. Find the probability of getting a "lucky" ticket.

Extra Problems

6. In a box there are 28 black balls and 4 white. 10 balls are chosen randomly from among them. What is the probability that, among the chosen balls

   (a) there is at least one white ball

   (b) there is exactly one white balls

   (c) there are at least two white balls

   (d) there are exactly two white balls

7. Find the probability that among 50 students attending a lecture on probability theory, at least two of them have the same date of birth.

8. A deck of playing cards contains 52 cards, divided into 4 different suits of 13 cards each. 6 cards are randomly drawn. Find the probability that

   (a) Among these cards there will be the king of spades.

   (b) Among these cards there will be a representative of each suit.

9. A set of n balls is randomly placed into m boxes. Find the probability that all boxes are non-empty if the balls are distinguishable.

10. A shelf holds 12 books in a row. We pick 5 books randomly. Find the probability that no pair of adjacent books is chosen.

11. What is most likely to happen? Getting 6 on at least one die when four dice are thrown simultaneously, or getting a doublet of 6 (6 in both dice) when two different dice are thrown simultaneously 24 times.

12. Two M-sided dice are thrown. Find the probability that the sum of the two numbers obtained is equal to i.