

Problem set 4 - Bernoulli scheme. Independence

1. It's given that $P(ABC) = P(A)P(B)P(C)$. Does it imply that events A, B and C are mutually independent?
2. Consider two coins. Let $A = \{\text{first coin lands heads}\}$, $B = \{\text{second coin lands heads}\}$, $C = \{\text{heads appeared only once}\}$. Are A, B and C mutually independent?
3. Consider the Bernoulli scheme. Let A_i be a random subset of $\{1, 2, \dots, n\}$. Consider m such sets. Find **a)** $P(A_i \cap A_j = \emptyset)$ for arbitrary i, j . **b)** $P(A_1 \subset A_2 \cap A_3)$.
4. There are a white and b black balls in a box. Someone takes balls from the box, one at a time. A_k is the event that, at time k , he picks a white ball. Are the events A_1, \dots, A_n independent?
5. Find the probability that in $2n$ trials of the Bernoulli scheme with probability of success p and failure $q = 1 - p$ there will be $m + n$ successes and all trials with even numbers will end in success.
6. In the Bernoulli scheme, the probability of success (getting 1) is $p < 1$. Find the probability that in an infinite sequence of trials:
 - (a) 00 will appear before 01;
 - (b) 00 will appear before 10.
7. Let A_1, \dots, A_n be mutually independent event. Prove that for every $k \leq n$, events $\overline{A_1}, \dots, \overline{A_k}, A_{k+1}, \dots, A_n$ are mutually independent as well.

Extra Problems

8. Three dice are thrown. Events A, B and C stand for rolling matching numbers (two 6s, for example) on the first and second, on the second and third, on the first and third dice, respectively. Are these events pairwise and mutually independent?
9. Consider the Bernoulli scheme. $A = \{\text{n-th turn is 1}\}$, $B = \{\sum_{i=1}^{n-1} a_i = k\}$, $a_j \in \{0, 1\}$. Show that events A and B are independent.
10. Consider the Bernoulli scheme. Let A_i be a random subset of $\{1, 2, \dots, n\}$. Consider m such sets. Find $P(|A_1 \cup \dots \cup A_m| = k)$.
11. We randomly order the numbers from 0 to 99, getting the sequence $(x_1, x_2, \dots, x_{100})$. Are the following events independent: $\langle x_{80} > x_{81} \rangle$ and $\langle x_{81} > x_{82} \rangle$.
12. A particle changes its position by one unit every second. It moves to the right with probability p and to the left with probability $1 - p$. Let x_t correspond to the position of the particle at time t . Find the probability that at time $t = n$ the particle moved k units to the right. Let $x_0 = 0$.

13. Let A_1, \dots, A_n be independent events. Prove that

$$\mathbf{P}\left(\bigcup_{i=1}^n A_i\right) = 1 - \prod_{i=1}^n \mathbf{P}(\overline{A_i})$$

14. Let A, B, C be pairwise independent equiprobable events, and $A \cap B \cap C = \emptyset$. Find the maximum possible value of $\mathbf{P}(A)$.