Problem set 5 - Conditional probability, Bayes Formula

1. Three dice are rolled. Find the probability of getting 6 on at least one die if

   a) The numbers shown on the three dice are different.

   b) The number shown on the first and on the third die is the same.

2. In box A there are 10 white balls and 20 black balls. In box B there are 10 white balls and 10 black balls. We randomly pick 4 balls out of box A and 6 balls out of box B and put them all together in an empty box C. What is the probability of picking a white ball from box C?

3. The exam in probability theory consists of n questions. k of these questions are "easy". Find the probability that in a group of n students **a)** The first student chooses an easy question. **b)** The ith student chooses an easy question.

4. Box A contains a white balls and b black balls. Box B contains c white balls and d black balls. From a randomly chosen box is taken a ball (without return), which turns out to be white. Find the probability that a second ball randomly taken from the same box will be white.

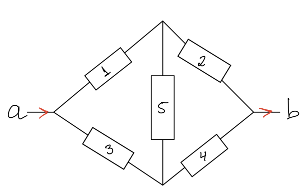
5. *(Monty Hall problem)* Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?"Is it to your advantage to switch your choice?

6. Two players conduct an endless series of independent tests. In each test, player A rolls 3 dice and player B rolls 2 dice at the same time. They carry out these tests until they obtain "six"on at least one of the dice. Find the probability of the following events:

   (a) The first "six" was rolled by player A, and not by B.

7. All tickets (100 tickets) for the Moscow-Berlin flight have been purchased. All passengers arrived for boarding. The first person to enter the plane is an old woman who sits in a random seat. Passengers enter in turn, and each next passenger takes his seat, if it is free. If the seat is occupied, then the passenger sits in a random seat from the remaining ones. What is the probability that the last passenger will take their seat?

8. The following circuit operates only if there is a path of functional devices from left to right (from a to b). The probability that device i functions is $\frac{i}{10}$. Assume that the devices fail independently. What is the probability devise 5 functions if the circuit operates?



Extra Problems

9. Two snipers shoot a target. Sniper A hits with probability 0.5 and sniper B hits with probability 0.8. They toss a fair coin to determine who shoots first. Find the probability that sniper A shot first if it is known that after the first shot the target was hit.

10. A patient goes to see a doctor. The doctor performs a test with 99 percent reliability–that is, 99 percent of people who are sick test positive and 99 percent of the healthy people test negative. The doctor knows that only 1 percent of the people in the country are sick. Now the question is: if the patient tests positive, what are the chances the patient is sick?

11. A man is known to speak the truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

12. Two players conduct an endless series of independent tests. In each test, player A rolls 3 dice and player B rolls 2 dice at the same time. They carry out these tests until they obtain "six"on at least one of the dice. Find the probability of the following events:

    (a) The first “six” was rolled by both A and B at the same time.

13. Two subsets A_1, A_2 are chosen randomly from $\{1, \dots, n\}$ (they may coincide). Find the probability that $|A_1| = \ell_1$, $|A_2| = \ell_2$ under the condition that they do not intersect.

14. There are 10 phones in a store, for which the probability of proper functioning within a month is 0.9 and 5 phones with a similar probability of 0.95. Find the probability that two phones, bought randomly from the store, will work properly for a month.

15. In a box there are N balls. M of them are white. We take out n balls and put them back. Consider the following events: $A_k = \{\text{in the k-th trial we took a white ball}\}$,

    $B_m = \{\text{we took out } m \text{ white balls in total}\}$. Find $\mathsf{P}(A_k|B_m)$.

16. From an urn containing a white and b black balls, two players take turns drawing balls. The winner is whoever chooses the white ball first. Find the probability that the first player wins in cases where the balls are drawn

    (a) under the scheme of equiprobable choice with return;

    (b) under the scheme of equiprobable choice without replacement.