Problem set 9 - Independence. Convolution formula

1. Let ξ_1, ξ_2 be independent random variables, uniformly distributed on $[0, 2]$. Find $\mathsf{P}(1 \leq \xi_2\xi_2 \leq 2)$.

2. Let ξ, η be independent random variables with uniform distribution in $[0, a]$. Find the density of the random variables (a) $\xi + \eta$, (b) $\xi\eta$.

3. Let X and Y be independent random variables. Find the probability that segments with lengths X, Y and 1 can form a triangle if X has uniform distribution on $[0, 1]$ and Y has exponential distribution with parameter 1.

4. Let ξ_1 and ξ_2 be independent random variables. $\mathsf{P}(\xi_1 = 0) = \mathsf{P}(\xi_1 = 1) = 1/2$. Random variable ξ_2 has uniform distribution on $[0, 1]$. Find the probability distribution of $\xi_1 + \xi_2$.

5. Let ξ_1 and ξ_2 be independent random variables. Find $p_{\xi_1+\xi_2}$ if

   (a) $\xi_i \sim \Gamma(\alpha_i, \lambda)$, $i = 1, 2$.

6. Let ξ_1 and ξ_2 be independent random variables with distribution $\mathsf{Exp}(1)$. Prove that the random variables $\eta_1 = \frac{\xi_1}{\xi_1+\xi_2}$ and $\eta_2 = \xi_1 + \xi_2$ are independent, and find their distributions.

7. Let $\xi_1, \ldots, \xi_n$ be independent random variables with standard normal distribution. Find that distribution of $\xi_1^2 + \ldots + \xi_n^2$.

Extra Problems

8. Let ξ, η be independent random variables with uniform distribution in $[0, a]$. Find the density of the random variables $\xi - \eta$, ξ/η.

9. Let ξ_1 and ξ_2 be independent random variables. Find $p_{\xi_1+\xi_2}$ if

   (a) $\xi_i \sim \mathcal{N}(a_i, \sigma_i^2)$, $i = 1, 2$;

10. Let $\xi_1, \xi_2, \ldots, \xi_n$ be a sequence of independent random variables with exponential distribution with parameter λ. Find the PDF of the random variable $S_n = \xi_1 + \ldots + \xi_n$.

11. Let ξ_1 and ξ_2 be random variables with standard normal distribution. Is it always true that $\xi + \eta$ has standard normal distribution?

12. Let ξ, η be independent absolutely continuous random variables. Find the density of the random vector $(\xi, \xi + \eta)$.

13. Let ξ_1, ξ_2, ξ_3 be i.i.d. random variables with uniform distribution in $[0, 1]$. Find the density of the random variable $\xi_1 + \xi_2 + \xi_3$.