List of Exam Questions - Probability Theory I by *Carlos Buitrago*

1. Basics of combinatorics. Rule of sum and rule of product. Pigeon-hole principle. Permutations and combinations. Binomial and Multinomial theorems. Combinatorial identities (7 formulas). [*pag 8-21 from* [1]]

2. Inclusion-exclusion principle. Random experiments and notion of probability. Probability in nature. Probability triple $(\Omega, \mathcal{F}, \mathrm{P})$ [*pag 58 from* [2]]. Discrete sample space. Classical probability model. Problem on getting a lucky ticket in a tram in Dolgoprudny.

3. Bernoulli scheme. k successes in n trials. Sum of all outcomes gives 1. Geometric probability. Probability of two people meeting. Bertrand's Paradox.

4. Probability of union of intersecting events and its upper bound [*pag 25 from* [1]]. Conditional probability. Theorem of multiplication. Independence of events (pairwise and mutually). Example of pairwise but not necessarily mutually independent events. Example on the necessary conditions for mutual independence. [*pag 63 from* [1]]

5. Partition of Ω. Total Probability Formula. Bayes Theorem [*pag 46-58 from* [1]]. Monty-Hall problem. Example with blood test. Problem on choosing an easy question at the exam. Problem of an old lady in a plane. Notion of random variables [*pag 103 from* [1]]. Number of successes in Bernoulli scheme as a random variable [*pag 112 from* [1]].

6. Random variables as a measurable function [*pag 170 from* [3]]. Probability Mass Function (PMF) of a random variable. Sum of independent random variables with Binomial distribution. Sum of independent random variables with Poisson distribution [*pag 131, 181 from* [1]].

7. Independence of discrete random variables [*pag 129 from* [1]]. Examples of discrete distributions ($\mathsf{Ber}(p)$, $\mathsf{Bin}(n,p)$, $\mathsf{Geom}(p)$, $\mathsf{Pois}(\lambda)$). Poisson limit theorem [*pag 64 from* [3]].

8. Expected value of a random variable [*pag 149 from* [1]]. Properties of the Expected value [*pag 170 from* [1]]. Variance. Properties with proofs (3) [*pag 171 from* [1]]. Covariance [*pag 326 from* [1]]. Example of dependent random variables with zero covariance.

9. Expected value and Variance of random variables with distribution $\mathsf{Ber}(p)$, $\mathsf{Bin}(n,p)$, $\mathsf{Geom}(p)$, $\mathsf{Pois}(\lambda)$ [*pag 156-160 from* [1]]. Properties of covariance (5). Variance of sum of n random variables. Correlation coefficient, definition and meaning. $|\mathsf{corr}(\xi,\eta)| \leq 1$. Proof using variance. [*pag 326-331 from* [1]]

10. Weak Law of large numbers. General form of WLLN. [*pag 467 from* [1]]

11. Galton-Watson branching process. Probability of extinction. Distribution of individual in generation n. Probability generating funcion with properties.

12. De Moivre-Laplace Local limit Theorem. De Moivre-Laplace Integral limit Theorem. Finding the smallest number of coin tosses, at which the frequency of the appearance of Heats differs from $\frac{1}{2}$ no more than by $0,1$ with probability at least $0,9$. [*pag 55-64 from* [3]]

Список литературы

[1] Introduction To Probability - Joseph K. Blitzstein, Jessica Hwang.

[2] Introduction to probability for Data Science - Stanley H. Shan.

[3] Probability (Graduate Texts in Mathematics) 2nd Edition - Albert N. Shiryaev.

[4] Simple random walks by Sven Erick Alm (chapter 4)
Link: *http://www2.math.uu.se/ sea/kurser/stokprocmn1/slumpvandring$_e$ng.pdf*